

Contents lists available at [ScienceDirect](http://ScienceDirect)**Physics Letters B**[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

# Fermion tunnels of higher-dimensional anti-de Sitter Schwarzschild black hole and its corrected entropy

Kai Lin<sup>\*</sup>, ShuZheng Yang*Institute of Theoretical Physics, China West Normal University, NanChong, SiChuan 637002, China*

## ARTICLE INFO

### Article history:

Received 26 April 2009

Received in revised form 18 August 2009

Accepted 11 September 2009

Available online 18 September 2009

Editor: M. Cvetič

### PACS:

04.70.Dy

04.62.+v

03.65.Sq

### Keywords:

Higher-dimensional anti-de Sitter

Schwarzschild black hole

Corrected entropy

Fermions tunneling

## ABSTRACT

Applying the method beyond semiclassical approximation, fermion tunneling from higher-dimensional anti-de Sitter Schwarzschild black hole is researched. In our work, the “tortoise” coordinate transformation is introduced to simplify Dirac equation, so that the equation proves that only the  $(r - t)$  sector is important to our research. Because we only need to study the  $(r - t)$  sector, the Dirac equation is decomposed into several pairs of equations spontaneously, and we then prove the components of wave functions are proportional to each other in every pair of equations. Therefore, the suitable action forms of the wave functions are obtained, and finally the correctional Hawking temperature and entropy can be determined via the method beyond semiclassical approximation.

Crown Copyright © 2009 Published by Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/3.0/).

## 1. Introduction

The prospect of Hawking radiation from black holes has drawn researchers' attention [1,2], and people have proved this radiation via several methods. Recently, Parikh and Wilczek proposed tunneling theory to explain and research Hawking radiation of black holes [3,4]. In their theory, the radiation results from the quantum tunneling effect: the virtual particle tunnel from inside the black holes' horizon to the outside, where the particle can materialize to become a real particle. Subsequently, the semiclassical Hamilton–Jacobi method is proposed to effectively study the scalar particle tunneling. In 2007, Kerner and Mann proposed the semiclassical method to research fermion tunneling on several 5-dimensional, 4-dimensional and lower-dimensional cases [5,6]. In this method, the up-spin and down-spin cases are respectively researched and the correct tunneling rate and Hawking temperature were determined. We then expanded the method to derive the semiclassical Hamilton–Jacobi equation from the Dirac equation and study fermion tunneling from several higher-dimensional black holes [7].

On the other hand, in 2008 Banerjee and Majhi proposed a method beyond semiclassical approximation to research quantum

tunneling at the horizon [8,9]. In their work, all quantum corrections are computed, and the results show that semiclassical conclusions need to be corrected. Especially, the correctional entropy from this method is in accordance with the conclusions of recent quantum gravitation research. Here we will develop the method to research on fermion tunneling from higher-dimensional anti-de Sitter Schwarzschild black hole. We will apply the “tortoise” coordinate transformation to prove that only the  $(r - t)$  sector is effectively important in this research. In the  $(r - t)$  sector Dirac equation, the Dirac equation set is decomposed into several pairs of equations spontaneously, and we then prove the components of wave functions are proportional to each other in every pair of equations. Next, we rewrite the wave function as suitable action forms, and calculate every pair of equations via the method beyond semiclassical approximation. Finally, we obtain the correctional Hawking temperature and entropy.

## 2. Dirac equation in higher-dimensional anti-de Sitter Schwarzschild space–time

In higher-dimensional theory, the research can help to develop several modern physical theories [10], the metric of  $n$ -dimensional anti-de Sitter Schwarzschild space–time is given by [11,15]

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{n-2}^2, \quad (1)$$

<sup>\*</sup> Corresponding author.E-mail addresses: [lk314159@126.com](mailto:lk314159@126.com) (K. Lin), [szyangcwnu@126.com](mailto:szyangcwnu@126.com) (S.Z. Yang).

where

$$f(r) = 1 - \left( \frac{16\pi G_n M}{(n-2)\Omega_{n-2} r^{n-3}} + \frac{r^2}{l^2} \right) \quad (2)$$

(in this equation,  $G_n$  is  $n$ -dimensional Newton constant;  $d\Omega_{n-2}^2$  is the metric of unit  $S^{n-2}$ ;  $\Omega_{n-2}$  is the area of the unit sphere; the  $n$ -dimensional cosmological constant  $\Lambda = -(n-1)(n-2)/2l^2$ ). The event horizon's position  $r_h$  should be determined by the equation  $f(r_h) = 0$ . In this space-time, we can take the higher-dimensional Dirac equation into account, namely

$$\gamma^\mu D_\mu \Psi + \frac{m}{\hbar} \Psi = 0, \quad \mu = t, r, \dots, \eta, \dots, \quad (3)$$

where  $\eta$  are extra-dimensional coordinates and angular coordinates, and

$$D_\mu = \partial_\mu + \frac{i}{2} \Gamma_{\mu}^{\alpha\beta} \Pi_{\alpha\beta}, \quad (4)$$

$$\Pi_{\alpha\beta} = \frac{i}{4} [\gamma_\alpha, \gamma_\beta]. \quad (5)$$

In the quantum equation, the gamma matrices satisfy the relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I. \quad (6)$$

In  $n$ -dimensional space-time, we can choose gamma matrices like these

$$\gamma_{m \times m}^t = \frac{i}{\sqrt{f}} \hat{\gamma}_{m \times m}^1 = \frac{i}{\sqrt{f}} \begin{pmatrix} I_{\frac{m}{2} \times \frac{m}{2}} & 0 \\ 0 & -I_{\frac{m}{2} \times \frac{m}{2}} \end{pmatrix}, \quad (7)$$

$$\gamma_{m \times m}^r = \sqrt{f} \hat{\gamma}_{m \times m}^2 = \sqrt{f} \begin{pmatrix} 0 & I_{\frac{m}{2} \times \frac{m}{2}} \\ I_{\frac{m}{2} \times \frac{m}{2}} & 0 \end{pmatrix}, \quad (8)$$

...

$$\gamma_{m \times m}^\eta = \sqrt{g^{\eta\eta}} \hat{\gamma}_{m \times m}^\eta = \sqrt{g^{\eta\eta}} \begin{pmatrix} 0 & -i \hat{\gamma}_{\frac{m}{2} \times \frac{m}{2}}^{\eta-2} \\ i \hat{\gamma}_{\frac{m}{2} \times \frac{m}{2}}^{\eta-2} & 0 \end{pmatrix}, \quad \eta \geq 3, \quad (9)$$

where  $m = 2^{n/2}$  ( $m = 2^{(n-1)/2}$ ) is the order of the matrix in even (odd) dimensional space-time;  $I_{\frac{m}{2} \times \frac{m}{2}}$  is a unit matrix with  $\frac{m}{2} \times \frac{m}{2}$  orders;  $\gamma_{\frac{m}{2} \times \frac{m}{2}}^\nu$  and  $\hat{\gamma}_{\frac{m}{2} \times \frac{m}{2}}^\nu$  are the  $\nu$ -th gamma matrix with  $\frac{m}{2} \times \frac{m}{2}$  orders in curved and flat space-time, respectively (in flat space-time, the gamma matrices should satisfy the anti-commutation relation  $\{\hat{\gamma}_{\frac{m}{2} \times \frac{m}{2}}^\nu, \hat{\gamma}_{\frac{m}{2} \times \frac{m}{2}}^\mu\} = 2\delta_{\nu\mu} I_{\frac{m}{2} \times \frac{m}{2}}$ ). Specifically the  $2 \times 2$ -order flat gamma matrices are

$$\hat{\gamma}_{2 \times 2}^1 = \sigma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (10)$$

$$\hat{\gamma}_{2 \times 2}^2 = \sigma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (11)$$

$$\hat{\gamma}_{2 \times 2}^3 = \sigma^3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (12)$$

They are none other than the Pauli matrices. So the Dirac equation in this space-time can be rewritten as

$$i \hat{\gamma}^1 \frac{\partial \Psi}{\partial t} + \hat{\gamma}^2 \left( f \frac{\partial}{\partial r} - \frac{f'}{4} \right) \Psi + \sqrt{f} \sum_{i=3}^n \gamma^\eta D_\eta \Psi + \sqrt{f} \frac{m}{\hbar} \Psi = 0, \quad (13)$$

where  $f' = \frac{df}{dr}$ . Now, we need to prove that only  $(r-t)$  sector is important in the research about tunneling radiation at the horizon. We introduce the "tortoise" coordinate transformation as

$dr_* = dr/f$  (note  $f \rightarrow 0$  at the event horizon  $r_h$ ), and the Dirac equation is simplified near the horizon as

$$i \hat{\gamma}^1 \frac{\partial \Psi}{\partial t} + \hat{\gamma}^2 \left( \frac{\partial}{\partial r_*} - \frac{f'}{4} \right) \Psi = 0. \quad (14)$$

The above equation shows that the property of fermion tunneling at the horizon is very simple. We can rewrite the equation as ordinary form

$$\frac{i \hat{\gamma}^1}{\sqrt{f}} \frac{\partial \Psi}{\partial t} + \sqrt{f} \hat{\gamma}^2 \left( \frac{\partial}{\partial r} - \frac{f'}{4f} \right) \Psi = 0. \quad (15)$$

In this space-time,  $D_t = \frac{\partial}{\partial t} + \frac{f'}{4} \gamma^t \gamma^r$  and  $D_r = \frac{\partial}{\partial r}$ , so Eq. (15) is none other than the  $(r-t)$  sector of the Dirac equation, and we just research the radial part  $\psi(r, t)$  of  $\Psi$ . Next, we need to look at the wave function  $\psi$ . In static space-time,  $\frac{\partial}{\partial t} \leftrightarrow -i\omega$  corresponds to a Killing vector, so the radial wave function can be rewritten as [12]

$$\psi = \begin{bmatrix} A(r) \\ B(r) \end{bmatrix} e^{-\frac{i}{\hbar} \omega t}, \quad (16)$$

where  $A(r)$  and  $B(r)$  are matrices with  $\frac{m}{2} \times \frac{m}{2}$  elements, and  $\omega$  is the fermion energy. Therefore, the Dirac equation becomes

$$\begin{aligned} \omega A_q + \hbar f \left( \frac{\partial}{\partial r} - \frac{f'}{4f} \right) B_q &= 0, \\ -\omega B_q + \hbar f \left( \frac{\partial}{\partial r} - \frac{f'}{4f} \right) A_q &= 0, \quad q = 1, 2, \dots, m/2, \end{aligned} \quad (17)$$

where  $A_q$  and  $B_q$  are the  $q$ -th elements of the column matrices  $A(r)$  and  $B(r)$ . It shows that the Dirac equation set is decomposed into several pairs of equations spontaneously, so that we needn't differentiate the Dirac equation into up-spin and down-spin cases. From Eq. (17), we can obtain

$$\frac{\frac{\partial B_q}{\partial r}}{\frac{\partial A_q}{\partial r}} = \frac{\omega A_q - \hbar \frac{f' B_q}{4}}{-\omega B_q - \hbar \frac{f' A_q}{4}}. \quad (18)$$

Near the horizon, the equation is given by

$$\frac{\omega}{2} \frac{\partial}{\partial r} (A_q^2 + B_q^2) - \hbar \frac{f'}{4} \left( B_q \frac{\partial A_q}{\partial r} - A_q \frac{\partial B_q}{\partial r} \right) = 0. \quad (19)$$

Because  $f'$  depends on the position  $r_0$  of horizon, from the equation above, we can obtain the formulas

$$\begin{aligned} \frac{\partial}{\partial r} (A_q^2 + B_q^2) &= 0, \\ B_q \frac{\partial A_q}{\partial r} - A_q \frac{\partial B_q}{\partial r} &= 0. \end{aligned} \quad (20)$$

From Eq. (20), the relation between  $A_q$  and  $B_q$  is determined

$$A_q^2 + B_q^2 = 0. \quad (21)$$

At the horizon, our conclusion is in accordance with the results in Refs. [5,9]. In higher-dimensional anti-de Sitter Schwarzschild space-time, using this simple relation, we can research fermion tunneling beyond semiclassical approximation.

### 3. Fermion tunneling beyond semiclassical approximation

Near the horizon of this space-time, we can rewrite  $A_q(r)$  as action form

$$A_q(r) = C_q e^{\frac{i}{\hbar} R_q(r)}. \quad (22)$$

Due to the simple relation between  $A_q(r)$  and  $B_q(r)$ ,  $B_q(r)$  can be rewritten as

$$B_q(r) = F_q e^{\frac{i}{\hbar} R_q(r)}. \quad (23)$$

Observably, the relation between constants  $C_q$  and  $F_q$  is  $C_q = \pm i F_q$ . Therefore, the Dirac equation can be rewritten as

$$\begin{aligned} \frac{\omega}{\sqrt{f}} C_q + \sqrt{f} \left( i \frac{\partial R_q}{\partial r} - \hbar \frac{f'}{4f} \right) F_q &= 0, \\ -\frac{\omega}{\sqrt{f}} F_q + \sqrt{f} \left( i \frac{\partial R_q}{\partial r} - \hbar \frac{f'}{4f} \right) C_q &= 0. \end{aligned} \quad (24)$$

In WKB approximation, we can decompose action and energy of 1/2 spin particle as

$$R_q(r) = \sum_{i=0}^{\infty} \hbar^i R_{qi}(r), \quad (25)$$

$$\omega = \sum_{i=0}^{\infty} \hbar^i \omega_i, \quad (26)$$

where  $R_{q0}$  and  $\omega_0$  are semiclassical action and energy of fermion. Substituting (25) and (26) into (24) and then equating the different powers of  $\hbar$  on both sides, we can get the following equations

$$\hbar^0: \begin{pmatrix} -i \frac{\omega_0}{f} & \frac{\partial R_{q0}}{\partial r} \\ \frac{\partial R_{q0}}{\partial r} & i \frac{\omega_0}{f} \end{pmatrix} \begin{pmatrix} C_q \\ F_q \end{pmatrix} = 0, \quad (27)$$

$$\hbar^1: \begin{pmatrix} -i \frac{\omega_1}{f} & \frac{\partial R_{q1}}{\partial r} + \frac{if'}{4f} \\ \frac{\partial R_{q1}}{\partial r} + \frac{if'}{4f} & i \frac{\omega_1}{f} \end{pmatrix} \begin{pmatrix} C_q \\ F_q \end{pmatrix} = 0, \quad (28)$$

$$\hbar^k: \begin{pmatrix} -i \frac{\omega_k}{f} & \frac{\partial R_{qk}}{\partial r} \\ \frac{\partial R_{qk}}{\partial r} & i \frac{\omega_k}{f} \end{pmatrix} \begin{pmatrix} C_q \\ F_q \end{pmatrix} = 0, \quad k \geq 2. \quad (29)$$

In every set of equations, we can obtain two possible solutions

$$\begin{aligned} \hbar^0: \quad C_q &= -i F_q, \quad R_{q0+} = \int \frac{\omega_0}{f} dr, \\ C_q &= i F_q, \quad R_{q0-} = - \int \frac{\omega_0}{f} dr, \end{aligned} \quad (30)$$

$$\begin{aligned} \hbar^1: \quad C_q &= -i F_q, \quad R_{q1+} = \int \frac{\omega_1 - \frac{if'}{4}}{f} dr, \\ C_q &= i F_q, \quad R_{q1-} = - \int \frac{\omega_1 - \frac{if'}{4}}{f} dr, \end{aligned} \quad (31)$$

$$\begin{aligned} \hbar^0: \quad C_q &= -i F_q, \quad R_{qk+} = \int \frac{\omega_k}{f} dr, \\ C_q &= i F_q, \quad R_{qk-} = - \int \frac{\omega_k}{f} dr, \quad k \geq 2, \end{aligned} \quad (32)$$

where  $R_{qi+}$  and  $R_{qi-}$  are radial outgoing and ingoing action respectively, and the total action is given by

$$R_{qi} = R_{qi+} - R_{qi-} = 2\omega_i \int \frac{dr}{f}, \quad i = 0, 1, 2, 3, \dots \quad (33)$$

The total action form is the same, so the solutions of Eqs. (30)–(32) are not independent, and each total radial action is proportional to semiclassical radial action  $R_{q0}$ , namely,

$$R_q(r) = R_{q0}(r) + \sum_{i=1}^{\infty} \hbar^i R_{qi}(r) = \left( 1 + \sum_{i=1}^{\infty} \xi_i \hbar^i \right) R_{q0}(r), \quad (34)$$

where it is obvious that the dimensions of un-determined coefficients  $\xi_i$  are  $\hbar^{-i}$ . In units  $G = c = k_B = 1$ , we therefore rewrite the radial action as

$$\begin{aligned} R_q(r) &= \left( 1 + \sum_{i=1}^{\infty} \beta_i \frac{\hbar^{i-1}}{S_{BH}^i} \right) R_{q0}(r) \\ &= \frac{2\pi i \omega_0}{f'(r_h)} \left( 1 + \sum_{i=1}^{\infty} \beta_i \frac{\hbar^{i-1}}{S_{BH}^i} \right), \end{aligned} \quad (35)$$

where  $\beta_i$  are dimensionless constants and  $S_{BH}$  is semiclassical Bekenstein–Hawking entropy. The correctional fermion tunneling rate is obtained

$$\begin{aligned} \Gamma_h &= \exp \left[ -\frac{2}{\hbar} \left( 1 + \sum_i \beta_i \frac{\hbar^{i-1}}{S_{BH}^i} \right) \text{Im} R_{q0}(r) \right] \\ &= \exp \left[ \frac{-4\pi i \omega_0}{\hbar f'(r_h)} \left( 1 + \sum_i \beta_i \frac{\hbar^{i-1}}{S_{BH}^i} \right) \right], \end{aligned} \quad (36)$$

and the correctional Hawking temperature is given by

$$T_h = \left( 1 + \sum_i \beta_i \frac{\hbar^{i-1}}{S_{BH}^i} \right) T_H, \quad (37)$$

where  $T_H$  is semiclassical Hawking temperature, which is given by

$$T_H = \frac{\hbar}{4\pi} f'(r_h). \quad (38)$$

Here we obtained the correctional Hawking temperature, and the results support the fermion tunneling work of Kerner, Mann and Majhi [5,9]. We will then research the correctional entropy at the horizon. In black hole thermodynamics, the famous law is [1,13]

$$dM = T_h dS_{bh} + \Theta dQ + \Omega dJ, \quad (39)$$

where  $\Theta$ ,  $Q$ ,  $J$  and  $\Omega$  are black holes' electromagnetic potential, electric charge, angular momentum and angular velocity respectively. It is clear that the correctional entropy of higher-dimensional anti-de Sitter Schwarzschild black hole is  $dS_{bh} = dM/T_h$ , so we can obtain

$$\begin{aligned} S_{bh} &= \int dS_{bh} = \int \frac{dM}{T_h} \\ &= S_{BH} + \beta_1 \ln S_{BH} + \text{const} + \dots \end{aligned} \quad (40)$$

In this space-time, when  $n = 4$ , we will get the correctional entropy of anti-de Sitter Schwarzschild black hole. It is apparent that the first correction term is no other than logarithmic corrections, and the fact squares with the study about correctional Bekenstein–Hawking entropy in quantum gravitation theory [14,15]. According to Ref. [15], the  $\beta_1$  of anti-de Sitter Schwarzschild black hole is  $-\frac{n}{2(n-2)}$ . This shows that the method beyond semiclassical approximation is correct. What's more, contrasting the method with other approach, this method is very simple and clear.

#### 4. Conclusions

In this Letter, we generalized the method beyond semiclassical approximation to research fermion tunneling from higher-dimensional anti-de Sitter Schwarzschild black hole, and obtained correctional Bekenstein–Hawking entropy. In our method, Eqs. (17)–(21) are very important, because they allow us to decompose the Dirac equation into forms which only involve single particle action as a derivative. Using this method, we can also study fermion tunneling from other static higher-dimensional black holes. What we will do next is to generalize the method to research stationary and non-stationary cases.

## Acknowledgement

This work is supported by National Natural Science Foundation of China (No. 10773008).

## References

- [1] S.W. Hawking, *Nature* 248 (1974) 30;  
S.W. Hawking, *Commun. Math. Phys.* 43 (1975) 199.
- [2] S.P. Robinson, F. Wilczek, *Phys. Rev. Lett.* 95 (2005) 011303, arXiv:gr-qc/0502074;  
T. Damour, R. Ruffini, *Phys. Rev. D* 14 (1976) 332;  
S. Sannan, *Gen. Relativ. Gravit.* 20 (1988) 239.
- [3] P. Kraus, F. Wilczek, *Nucl. Phys. B* 433 (1995) 403, arXiv:gr-qc/9408003;  
M.K. Parikh, F. Wilczek, *Phys. Rev. Lett.* 85 (2000) 5042, arXiv:hep-th/9907001.
- [4] S. Hemming, E. Keski-Vakkuri, *Phys. Rev. D* 64 (2001) 044006, arXiv:gr-qc/0005115;  
Q.Q. Jiang, S.Q. Wu, X. Cai, *Phys. Rev. D* 75 (2007) 064029;  
S. Iso, H. Umetsu, F. Wilczek, *Phys. Rev. D* 74 (2006) 044017, hep-th/0606018;  
A.J.M. Medved, *Phys. Rev. D* 66 (2002) 124009, arXiv:hep-th/0207247;  
M.K. Parikh, arXiv:hep-th/0402166;  
M.K. Parikh, J.Y. Zhang, Z. Zhao, *JHEP* 0510 (2005) 055;  
J.Y. Zhang, Z. Zhao, *Phys. Lett. B* 638 (2006) 110, arXiv:gr-qc/0512153;  
E.T. Akhmedov, V. Akhmedova, D. Singleton, *Phys. Lett. B* 642 (2006) 124;  
V. Akhmedova, T. Pilling, A. de Gill, D. Singleton, *Phys. Lett. B* 666 (2008) 269;  
K. Srinivasan, T. Padmanabhan, *Phys. Rev. D* 60 (1999) 24007;  
S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, *Class. Quantum Grav.* 19 (2002) 2671;  
M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, *J. High Energy Phys.* 0505 (2005) 014;  
S.Z. Yang, D.Y. Chen, *Int. J. Theor. Phys.* 46 (2007) 2923Y;  
S.Z. Yang, D.Y. Chen, *Chin. Phys. B* 17 (2008) 817.
- [5] R. Kerner, R.B. Mann, *Class. Quantum Grav.* 25 (2008) 095014, arXiv:0710.0612;  
R. Kerner, R.B. Mann, *Phys. Lett. B* 665 (2008) 277, arXiv:hep-th/0803.2246.
- [6] R. Li, J.R. Ren, S.W. Wei, *Class. Quantum Grav.* 25 (2008) 125016, arXiv:0803.1410;  
R. Li, J.R. Ren, *Phys. Lett. B* 661 (2008) 370, arXiv:0802.3954;  
D.Y. Chen, Q.Q. Jiang, X.T. Zu, *Class. Quantum Grav.* 25 (2008) 205022, arXiv:0803.3248;  
D.Y. Chen, Q.Q. Jiang, X.T. Zu, *Phys. Lett. B* 665 (2008) 106, arXiv:0804.0131;  
R.D. Criscienzo, L. Vanzo, *Europhys. Lett.* 82 (2008) 60001;  
L.H. Li, S.Z. Yang, T.J. Zhou, R. Lin, *Europhys. Lett.* 84 (2008) 20003;  
Q.Q. Jiang, *Phys. Lett. B* 666 (2008) 517;  
Q.Q. Jiang, *Phys. Rev. D* 78 (2008) 044009;  
K. Lin, S.Z. Yang, *Int. J. Theor. Phys.* 48 (2009) 2061.
- [7] K. Lin, S.Z. Yang, *Phys. Rev. D* 79 (2009) 064035;  
K. Lin, S.Z. Yang, *Phys. Lett. B* 674 (2009) 127.
- [8] R. Banerjee, R.B. Majhi, *JHEP* 0806 (2008) 095, arXiv:0805.2220 [hep-th];  
R. Banerjee, R.B. Majhi, *Phys. Lett. B* 662 (2008) 62;  
R. Banerjee, R.B. Majhi, S. Samanta, *Phys. Rev. D* 77 (2008) 124035;  
R. Banerjee, R.B. Majhi, *Phys. Rev. D* 79 (2009) 064024, arXiv:0812.0497 [hep-th];  
R. Banerjee, R.B. Majhi, D. Roy, arXiv:0901.0466 [hep-th];  
R.B. Majhi, S. Samanta, arXiv:0901.2258 [hep-th];  
S.K. Modak, *Phys. Lett. B* 671 (2009) 167, arXiv:0807.0959 [hep-th];  
R. Banerjee, R.B. Majhi, *Phys. Lett. B* 674 (2009) 218, arXiv:0808.3688 [hep-th];  
M. Siahann, A. Taiyanta, arXiv:0811.1132 [gr-qc];  
R.G. Cai, L.M. Cao, Y.P. Hu, *JHEP* 0808 (2008) 090, arXiv:0807.1232 [hep-th];  
J.Y. Zhang, *Phys. Lett. B* 668 (2008) 353, arXiv:0806.2441 [hep-th];  
R. Banerjee, S.K. Modak, *JHEP* 0905 (2009) 063, arXiv:0903.3321 [hep-th];  
T. Zhu, J.R. Ren, M.F. Li, arXiv:0906.4194 [hep-th];  
Y.Q. Yuan, X.X. Zeng, Z.J. Zhou, L.P. Jin, *Gen. Relativ. Gravit.*, doi:10.1007/s10714-009-0806-x;  
K. Lin, S.Z. Yang, *Europhys. Lett.* 86 (2009) 20006.
- [9] R.B. Majhi, *Phys. Rev. D* 79 (2009) 044005, arXiv:0809.1508 [hep-th].
- [10] R.A. Konoplya, A. Zhidenko, arXiv:0809.2822 [hep-th];  
R.A. Konoplya, A. Zhidenko, *Nucl. Phys. B* 777 (2007) 182, hep-th/0703231;  
F.R. Tangherlini, *Nuovo Cimento* 27 (1963) 636;  
R.C. Myers, M.J. Perry, *Ann. Phys.* 172 (1986) 304;  
S. Creek, O. Efthimiou, P. Kanti, K. Tamvakis, *Phys. Lett. B* 635 (2006) 39, hep-th/0601126.
- [11] G.T. Horowitz, V.E. Hubeny, *Phys. Rev. D* 62 (2000) 024027;  
S. Hemming, E. Keski-Vakkuri, *Phys. Rev. D* 64 (2001) 044006.
- [12] S. Chandrasekhar, *Proc. R. Soc. London A* 349 (1976) 571;  
D.N. Page, *Phys. Rev. D* 14 (1976) 1509.
- [13] J.D. Bekenstein, *Lett. Nuovo Cimento* 4 (1972) 737;  
J.D. Bekenstein, *Phys. Rev. D* 7 (1973) 2333.
- [14] S.W. Hawking, *Commun. Math. Phys.* 55 (1977) 133;  
R.K. Kaul, P. Majumdar, *Phys. Rev. Lett.* 84 (2000) 5255;  
D.V. Fursaev, *Phys. Rev. D* 51 (1995) R5352;  
S. Mukherjee, S.S. Pal, *JHEP* 0205 (2002) 026;  
V.P. Frolov, W. Israel, S.N. Solodukhin, *Phys. Rev. D* 54 (1996) 2732;  
M.R. Setare, *Phys. Lett. B* 573 (2003) 173;  
M.R. Setare, *Eur. Phys. J. C* 38 (2004) 389.
- [15] S. Das, P. Majumdar, R.K. Bhaduri, *Class. Quantum Grav.* 19 (2002) 2355.